Homework 9

P8.1.6 Given **A** = 3 + j5, **B** = 10 – j8, and **C** = j12. Determine the phasors resulting from the following operations: (a) **A**∗**B**∗**C**; (b) (**A**∗**B**)/**C**; (c) (**A**/**B**)∗**C**; and (d) **A**/**B**/**C**. Express the result in rectangular and polar forms.

Solution: **A** = 3 + j5 = 5.83∠59.04°

B = 10 – j8 = 12.81∠-38.66°

C = j12 = 12∠90°

(a) **A**∗**B**∗**C** = 896.1∠110.4° ≡ -312 + j840

(b) (**A**∗**B**)/**C** = 6.22∠-69.62° ≡ 2.17 – j5.83

(c) (**A**/**B**)∗**C** = 5.46∠187.7° ≡ -5.41 – j0.73

(d) **A**/**B**/**C** = 0.0376 + j0.0051, 0.0379∠7.696°

P8.2.20 Determine Z_{Th} looking into terminals 'ab' in Figure P8.2.20.

Solution: The admittance of L in parallel with C -j/20 + j/10 = j0.05 S. It follows that $\mathbf{V_X} = 2\mathbf{V_X}\times j0.05$, so that $\mathbf{V_X}$ = 0. Hence, Z_{Th} = 1/j0.05 = -j20 Ω.

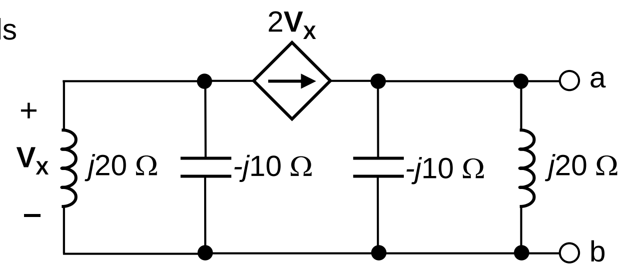


Figure P8.2.20

P8.3.12 Determine $\mathbf{V_X}$ in Figure P8.3.12.

Solution: The j2 Ω in series with -j2 Ω is a short circuit. It follows that $\mathbf{V_X}$ =

$$= 20\angle 60^\circ \frac{2}{2+j2} = 10\angle 60^\circ (1-j) =$$

$$10\sqrt{2}\angle 15^\circ \text{ V.}$$

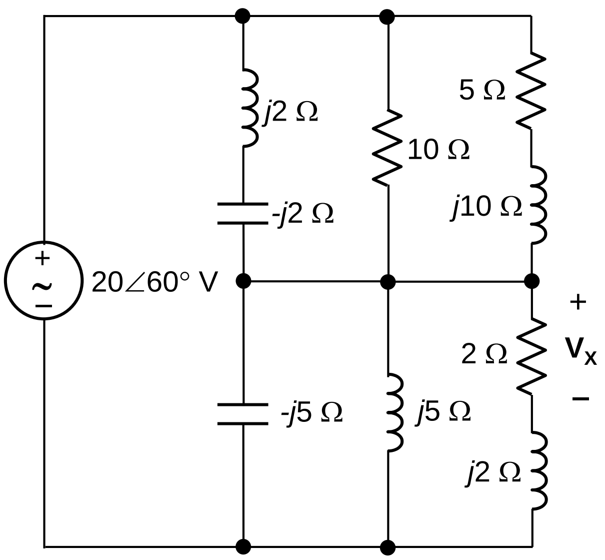


Figure P8.3.12

P8.3.26 Determine $i_C(t)$ in Figure P8.2.26, assuming $v_{SRC}(t)$ = sin(1000t + 30°) V.

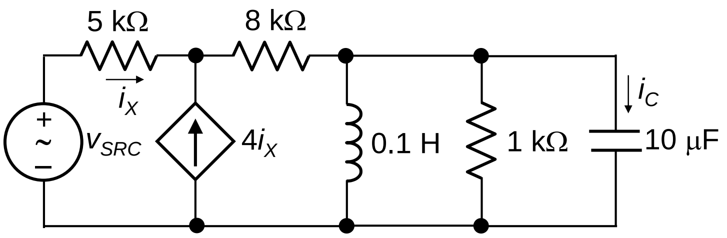
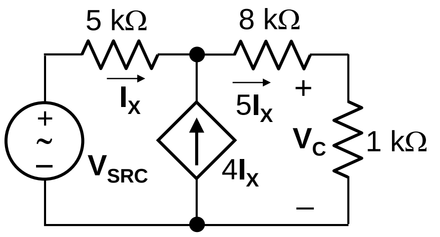


Figure P8.3.26

Solution: $j\omega L = j1000\times0.1 = j100\ \Omega$; $-j/\omega C = -j/(1000\times10^{-5}) = -j100\ \Omega$; it follows that the parallel impedance of L and C is infinite. The circuit reduces to that shown in the frequency domain. From KVL, $\mathbf{V_{SRC}} = 5\mathrm{I_X} + 5(8 + 1)\mathbf{I_X} = 50\mathbf{I_X}$; $\mathbf{I_X} = \mathbf{V_{SRC}}/50$ mA; $\mathbf{V_C} = 1(5\mathbf{I_X}) = \mathbf{V_{SRC}}/10$ V; $\mathbf{I_C} = j\omega C\mathbf{V_C} = j\mathbf{V_C}/100$ A $\equiv j\mathbf{V_{SRC}}$ mA $= j\times1\angle30° = 1\angle120°$ mA. Hence, i_C(t) = sin(1000t + 120°) mA.



P8.3.26-1

P8.4.10 Determine *Z* in figure P8.4.10 so that $\mathbf{V_o} = 1\angle\text{-}90°$ A.

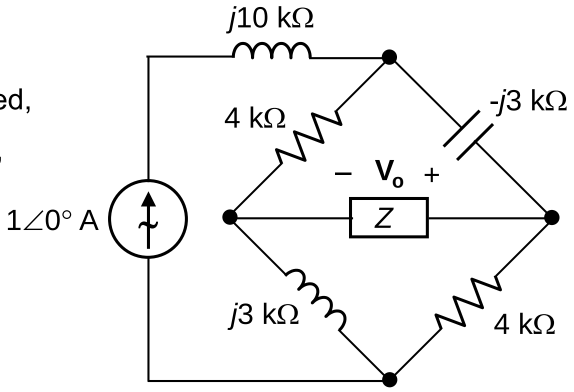


Figure P8.4.10

Solution: When terminals ‘ab’ are open circuited, $\mathbf{V_{Th}} = \mathbf{V_{bc}} - \mathbf{V_{ac}}$; from current division,

$$\mathbf{I_{ac}} = \frac{(4 - j3)}{8}, \text{ and}$$

$$\mathbf{V_{ac}} = \frac{j3(4 - j3)}{8};\ \mathbf{I_{bc}} = \frac{(4 + j3)}{8}, \text{ and}$$

$$\mathbf{V_{bc}} = \frac{4(4 + j3)}{8}; \text{ it follows that } \mathbf{V_{Th}} =$$

$$\mathbf{V_{ba}} = \frac{4(4 + j3)}{8} - \frac{j3(4 - j3)}{8} = \frac{7}{8}\text{ V.}$$

The impedance looking into terminals ‘ab’, with the current source set to zero, is $(4 - j3)||(4 + j3) = 25/8\ \Omega$. The circuit reduces to that shown. When $\mathbf{V_o} = 1\angle\text{-}90° = -j$ V, it follows from voltage division that $\mathbf{V_o} = -j = \frac{7}{8}\frac{Z}{Z + 25/8}$, Solving for *Z* gives: $Z = -\frac{25(8 + j7)}{113}\ \Omega$.

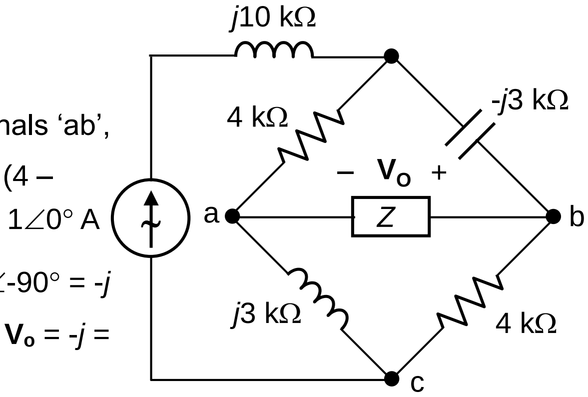


Figure P8.4.10-1

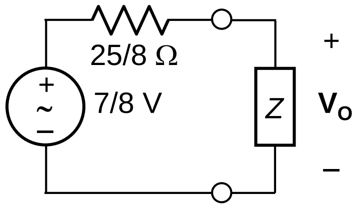


Figure P8.4.10-2

P8.4.11 Derive NEC looking into terminals 'ab' in Figure P8.4.11.

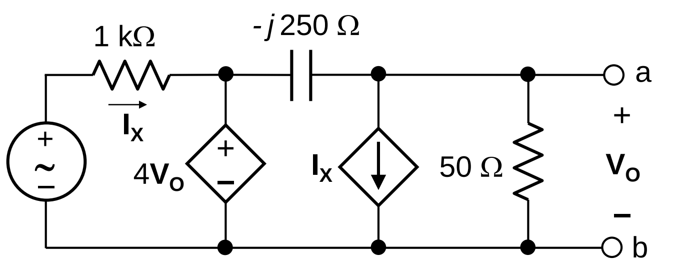


Figure P8.4.11

Solution: KCL at the right node:

$$\frac{\mathbf{V_o} - 4\mathbf{V_o}}{-j250} + \mathbf{I_x} + \frac{\mathbf{V_o}}{50} = 0, \text{ where } \mathbf{I_x} = \frac{100 - \mathbf{V_o}}{1000}.$$

Substituting, $\frac{3\mathbf{V_o}}{j250} + \frac{100 - 4\mathbf{V_o}}{1000} + \frac{\mathbf{V_o}}{50}\mathbf{V_o} = 0.$

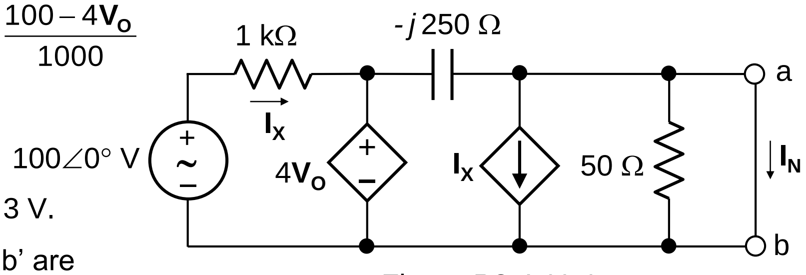


Figure P8.4.11-1

This gives $\mathbf{V_o} = -4 - j3$ V.

When terminals 'ab' are short-circuited, the dependent voltage source is set to zero, which makes $\mathbf{I_X}$ = 0.1 A. No current flows in the capacitor or in the 50 Ω resistor. It follows that $\mathbf{I_N} = -\mathbf{I_\phi}$ = -0.1 A. Hence, $Y_N = \frac{0.1}{4 + j3} = \frac{1}{250}(4 - j3)$ S.

P8.5.6 Determine $\mathbf{I_o}$ in Figure P8.5.6 using the mesh-current method.

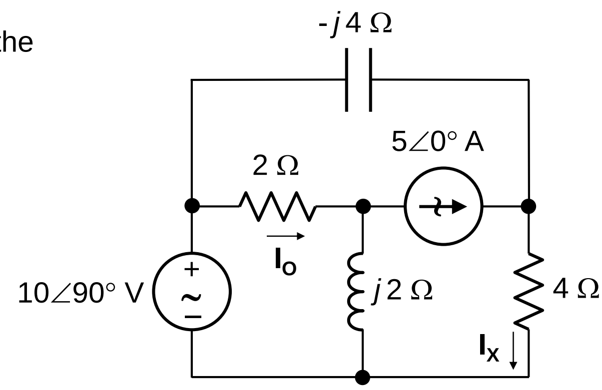


Figure P8.5.6

Solution: The mesh-current equations are:

Mesh 1: $(2 + j2)\mathbf{I_1} - j2\mathbf{I_2} - 2\mathbf{I_3} = j10$

Mesh 2: $-j2\mathbf{I_1} + (4 + j2)\mathbf{I_2} = \mathbf{V_y}$

Mesh 3: $-2\mathbf{I_1} + (2 - j4)\mathbf{I_3} = -\mathbf{V_y}$.

Adding:

$$-(2 + j2)\mathbf{I_1} + (4 + j2)\mathbf{I_2} + (2 - j4)\mathbf{I_3} = 0.$$

For the current source, $\mathbf{I_2} - \mathbf{I_3} = 5$. Solving these equations gives $\mathbf{I_o} = \mathbf{I_1} - \mathbf{I_3} =$ $5 + j5$ A.

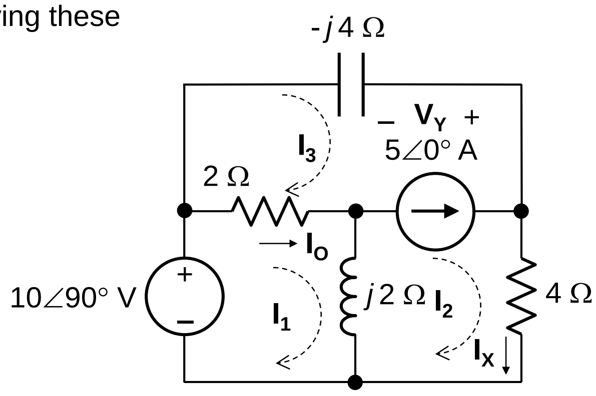


Figure P8.5.6-1

P8.5.7 Determine $v_c(t)$ in Figure P8.5.7 using the node-voltage method.

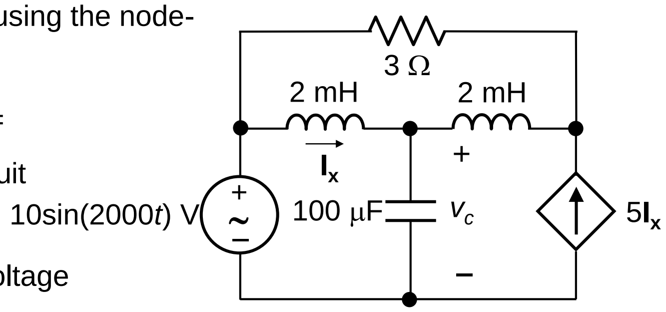


Figure P8.5.7

Solution: $\omega L = 2\times10^3\times2\times10^{-3} = 4\ \Omega$; $1/\omega C = 1/2\times10^3\times100\times10^{-4} = 5\ \Omega$; the circuit in the frequency domain becomes as shown. The node-voltage equations are:

Node 'a': $\mathbf{V_a} = 10$

Node 'b': $\mathbf{V_C}/\text{-}j5 + (\mathbf{V_C} - 10)/j4 + (\mathbf{V_C} - \mathbf{V_c})/j4 = 0$

Node 'c': $(\mathbf{V_c} - \mathbf{V_C})/j4 + (\mathbf{V_c} - 10)/3 = 5\mathbf{I_X} = (10 - \mathbf{V_C})/j4$. Solving these equations gives: $\mathbf{V_C} = 11.98 + j1.44 = 12.1\angle6.86°$, so that $v(t) = 12.1\sin(2000t + 6.86°)$ V.

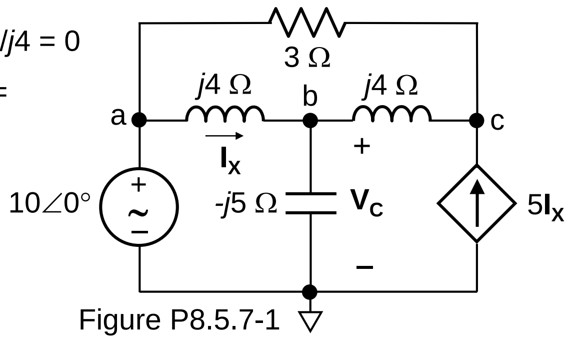


Figure P8.5.7-1